

One-dimensional analyses of striker impact on bar with different general impedance

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Hyunho Shin and Daesung Kim

Abstract

The particle velocity and stress in the striker and bar generated by the striker impact on a bar with different general impedance are formulated based on one-dimensional assumptions. Departure of the impact-generated stress wave towards the rear end of the striker and arrival of the release wave from the rear to the front of the striker constitute one impact cycle. In cases where $Z_s \leq Z_b$ (Z is the general impedance, and subscripts 's' and 'b' denote striker and bar, respectively), only one impact cycle takes place because the striker is stationary or separated from the impact surface after the first impact cycle. As a result, only a single (primary) pulse is observed in the bar and striker. In the case where $Z_s > Z_b$, however, multiple impact cycles take place because the striker is compressing the bar continually after the first cycle. As a result, a series of step-wise residual pulses follow the primary pulse in the bar and striker. The magnitudes of the stress and particle velocity in the bar and striker calculated using the formulated equations are quantitatively consistent with the results of the numerical simulations, verifying the formulated one-dimensional equations. The equations formulated in this study may be useful for better understanding the various wave interaction phenomena that take place in a pseudo-one-dimensional impact system and for modifying/designing an impact system.

Keywords

Stress, particle velocity, one-dimensional analysis, split Hopkinson bar, impact mechanics

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Introduction

There is great deal of interest in measuring dynamic material properties using pseudo-one-dimensional (1D) impact structures such as the split Hopkinson bar (SHB) system,^{1–9} direct impact test system,^{10–13} impact on rod system^{14,15} and Taylor impact test system.^{16,17} The SHB system in particular has been used extensively to measure uniaxial stress–strain and strain rate–strain curves of a wide range of materials^{18–22} at strain rates of approximately $10^2 - 10^4/\text{s}$. The dynamic properties measured by the aforementioned instruments are typically used for the calibration of strain-rate-dependent constitutive models,^{23–27} which are indispensable for the modelling and simulation of high-strain rate events.^{28–37}

The phenomena of stress wave generation, propagation, reflection and transmission in the mentioned pseudo-1D instruments can be revealed by numerical analyses. However, the equations developed based on 1D analyses are critical not only for measuring dynamic material properties but also in understanding/controlling various wave interaction

phenomena; they are also very important for designing/modifying the impact instruments themselves.

In the case of the SHB system, one of the most representative pseudo-1D impact instruments, a number of 1D equations were developed for measuring and controlling *specimen behaviour*. For instance, the processing equations of the SHB signals (incident, reflected and transmitted signals)^{1–9} used for determining the stress–strain and strain rate–strain curves of specimen are based on 1D assumptions. The second example includes the following 1D equations³ for the reflection ratio and transmission ratio of a pre-existing

Mechanics of Materials and Design Laboratory, Department of Materials Engineering, Gangneung-Wonju National University, Gangneung, Gangwon-do, Republic of Korea

Corresponding author:

Hyunho Shin, Mechanics of Materials and Design Laboratory, Department of Materials Engineering, Gangneung-Wonju National University, Gangneung, Gangwon-do 25457, Republic of Korea.
Email: hshin@gwnu.ac.kr

stress wave, which are used for understanding the increase of the specimen stress in the SHB test

$$\frac{\sigma_R}{\sigma_I} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad (1)$$

$$\frac{\sigma_T}{\sigma_I} = \frac{2A_1\rho_2C_2}{Z_1 + Z_2} \quad (2)$$

where σ_I is the magnitude of the *pre-existing* stress wave which is incident to a general interface, σ_R is the reflected stress from the interface, σ_T is the transmitted stress across the interface, Z is the general impedance ($Z = A\rho C$, where A is the cross-sectional area, ρ is the density and C is the speed of sound in a thin elastic rod) and subscripts 1 and 2 denote the media of the incident wave and transmitted wave, respectively. In terms of the particle velocity (U_p), the two ratios are as follows

$$\frac{U_{p,R}}{U_{p,I}} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \quad (3)$$

$$\frac{U_{p,T}}{U_{p,I}} = \frac{2Z_1}{Z_1 + Z_2} \quad (4)$$

Equations (1) and (2) were derived in Meyers,³ and equations (3) and (4) were derived here using backgrounds in Meyers.³ The last example of the 1D equation introduced here is the strain rate equation developed recently,⁹ which is very useful for designing SHB experiment and achieving a constant specimen strain rate in the standard SHB test.

As described above, the 1D equations of the SHB were primarily developed for measuring/controlling *specimen behaviour*, which are also very useful in other pseudo-1D impact instruments. Regarding the phenomena that occur in the *striker and bar* after their impact, only limited 1D equations have been developed.

It is known that, if the impedance (ρC) of the striker is higher than that of the bar in the compressive SHB, a series of step-wise residual stress pulses follow the primary stress pulse.⁷ In the case of the tensile SHB system,^{38–42} spurious waves can be observed after the primary pulse^{40,41} when there is the mismatch in material and/or diameter of the striker and flange.⁴² These phenomena occurring under the condition of a different general impedance across the impact surface cannot be properly understood/controlled using the existing 1D equations. For the purposes of not only understanding/controlling versatile impact phenomena including the mentioned examples, but also designing/modifying various pseudo-1D impact systems themselves, 1D equations describing the stress and particle velocity in the *striker and bar* are essential. The 1D equations for such purposes should be valid even when the cross-sectional areas and materials are different across the impact surface.

However, such 1D equations are rare in the literature compared with the equations developed for a special case where the cross-sectional area and material of the striker and bar are identical (which will be described later in this study).

Therefore, in this study, 1D analyses of the striker impact are conducted for the cases where general impedances are different across the impact surface. Equations of the impact-generated *stress* and *particle velocity* in the striker and bar will be formulated for these impact surfaces and verified by numerical simulation.

Method

Assumptions and sign convention

The 1D analyses in this study employ the following assumptions:

- i. The speed of the elastic wave in the striker and bar is described by a slender rod (1D) approximation ($C = \sqrt{E/\rho}$).
- ii. There is no length change of striker in spite of its impact on a circular bar.
- iii. The stress corresponding to the force equilibrium between the striker and bar is evenly distributed even in the region near the contact (impact) surface, where the edge of either the striker or bar is in contact with free space due to the difference in cross-sectional area (diameter).
- iv. There is no inertia in the striker and bar; the particle velocity and stress change immediately (at infinite acceleration) when the striker impacts the bar or wave interaction takes place.
- v. The stress pulse generated by the striker impact maintains its shape with its travel in the striker and bar. This assumption implies that the stress pulse is composed of waves with wavelengths that are much longer than the striker/bar diameter, so that the dispersion of the stress pulse is insignificant.

In this study, a positive sign is assigned to the compressive property for convenience. For the investigation of the particle velocity, it is also convenient to assign a positive sign to the direction of the striker flight. As the striker is drawn to move to the right in the figures in this study, a positive sign is assigned to a right-travelling wave.

Simulation method

To verify the formulated equations in this study, the phenomenon of striker impact on bars with different cross-sectional areas was simulated via an explicit finite element analysis. The diameter of the bar was 20 mm while that of the striker varied according to the cross-sectional area ratio of the striker to the

Table 1. Material properties and geometry of the striker and bar used for numerical simulation and 1D calculations.

Properties	Striker	Bar
Diameter, D (mm)	14.1421 ($A_s/A_b = 0.5$) 20.0000 ($A_s/A_b = 1.0$) 28.2843 ($A_s/A_b = 2.0$)	20
Length, L (mm)	300	2500
Poisson's ratio, ν	0.29	0.29
Density, ρ (kg/m ³)	7850	7850
Elastic modulus, E (GPa)	210	210
1D sound speed, $C (= \sqrt{E/\rho})$ (m/s)	5172.1942	5172.1942
General impedance, Z (kg/s)	6377.7039 ($A_s/A_b = 0.5$) 12755.4078 ($A_s/A_b = 1.0$) 25510.8156 ($A_s/A_b = 2.0$)	12755.4078

bar (A_s/A_b): 0.5, 1.0 and 2.0. Detailed geometries of the striker and bar are presented in Table 1. Considering the axial symmetry of the impact system, half of the two-dimensional axisymmetric geometry was discretised using four-node bilinear axisymmetric quadrilateral elements (CAX4R). The striker was initially 0.1 mm away from the bar, and its initial velocity was set as 15 m/s. The size of the element was approximately $1 \times 1 \text{ mm}^2$. The employed elements passed a mesh sensitivity test, which was performed separately. No friction was considered between the contact surfaces.

Structural steel was considered as the material for both the striker and the bar. The properties used in the simulation are listed in Table 1. Purely elastic behaviour of the material was assumed by not setting the yield criterion. A commercial finite-element package, ABAQUS, was used as the solver.

In the post processing stage, the particle velocity in the axial direction was extracted at nodes on the impact surfaces of the striker and the bar. The stresses were monitored by extracting the axial stress components of an element in the striker and bar, both at a distance 30 mm away from the impact surface. The reason these elements were selected instead of the elements at the impact surfaces was because, if the A_s/A_b ratio was not unity, the stress was not equally distributed particularly in the region near the contact (impact) surface, where the edge of the bar or striker was in contact with free space due to the difference in cross-sectional area (diameter).

In separate analyses of the quasi-static as well as dynamic striker loading on the bars with different area ratios (result not shown here), the magnitude of the stress coincided with the 1D analysis result from an approximately one diameter distance which was not far away from the contact (impact) surface. Such phenomena in quasi-static and dynamic loadings are examples of Saint Venant's principle of rapid dissipation of localized stress.⁴³

While this study considered the cases with different area ratios in the numerical simulation, the general

impedance ratio (Z_s/Z_b) was used in the discussion instead of the area ratio (A_s/A_b). In our separate analyses of the cases where the diameters were the same but impedance (ρC) ratios were different, the same findings and conclusion as the analysis for the different area ratios were drawn (the result is not shown here), justifying the use of the term, general impedance ($A\rho C$) in this study.

Impact-generated particle velocity and stress

Before investigating the impact-generated stress and particle velocity from a general impact surface, a special case where the diameter and material of the bar are the same as those of the striker is first considered as reference. Figure 1 shows a schematic of the considered case, where L is the striker length, V is the impact velocity, C is the sound speed in a thin elastic rod and U_p is the particle velocity, t is the time and Δt is the one-way travel time of the elastic stress pulse in the striker. In Figure 1, $t=0$ is the moment when the striker impacts the bar.

The particle velocity generated by the striker impact can be derived by considering the momentum conservation between the moments before ($t \leq 0$) and after the impact ($t = \Delta t$)

$$\rho A L V = \rho A (2L) U_p \quad (5)$$

where the left side is the momentum before the impact ($t \leq 0$) and the right side after the impact ($t = \Delta t$) at a moment when the stress pulse reaches the rear end of the striker. Equation (5) leads to

$$U_p = \frac{1}{2} V \quad (6)$$

which is also available in Meyers.³ In Meyers,³ equation (6) was derived by considering Newton's second law instead of momentum conservation. According to Meyers,³ the relationship between the stress (σ) and

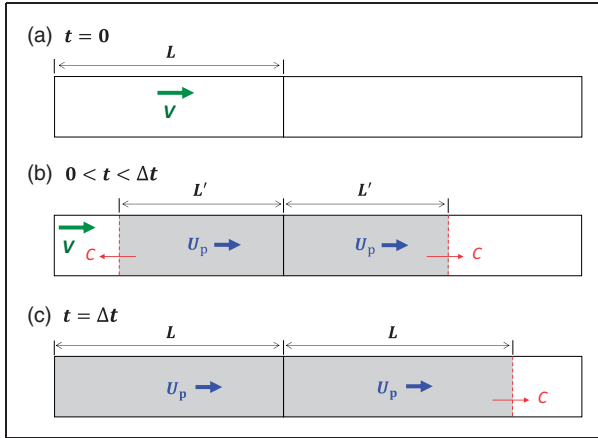


Figure 1. Momentum conservation in a special case where diameter and material of striker and the bar are identical. L is length of striker.

particle velocity (U_p) is $\sigma = \rho C U_p$ (which will also be derived later in equation (9) for a general impact surface). By applying equation (6) to this relationship, the stress (σ) in the bar and striker is given as a function of impact velocity:

$$\sigma = \rho C U_p = \frac{1}{2} \rho C V \quad (7)$$

The impact-generated particle velocity (U_p) and stress (σ) for the considered special case can be calculated from the impact velocity (V) using equations (6) and (7), respectively.

The stress build-up and particle velocity around the general impact surface are now considered. Figure 2 depicts a schematic of the considered case, where subscripts 's' and 'b' denote the striker and bar, respectively; and $\Delta t = L/C_s$. Figure 2(b) shows a moment during the travel of the impact-generated wave towards the rear end of the striker. At this moment, the particle velocities (U_p) in the stressed regions of the striker and bar are the same, which is the condition for the material continuity across the interface (interface continuity condition). Otherwise, the gap develops at the interface during the time period when the compressed (shaded) region in Figure 2(b), i.e. the primary pulse, is being generated from the impact surface.

In Figure 2(b), the required time ($\Delta t'$) for the stress wave to travel a distance of L' in the striker is $\Delta t' = L'/C_s$. The travel distance in the bar during the same time period $\Delta t'$ is $\Delta t' C_b$, which leads to $L' C_b / C_s$. Then, the mass (m_b) of the stressed region in the bar is $\rho_b A_b L' C_b / C_s$. The velocity change (dv_b) of the bar particles is $U_p - 0$ in the time interval of $\Delta t' (= L' / C_s)$. The force of the bar (F_b) is then

$$F_b = m_b \frac{dv_b}{dt} = \left(\rho_b A_b L' \frac{C_b}{C_s} \right) \left(\frac{U_p - 0}{L' / C_s} \right) = A_b \rho_b C_b U_p \quad (8)$$

from which the bar stress (σ_b) is

$$\sigma_b = \frac{F_b}{A_b} = \rho_b C_b U_p \quad (9)$$

Similarly, the mass (m_s) of the stressed region in the striker is $\rho_s A_s L'$. The velocity change (dv_s) of the striker particles is $V - U_p$ in the time interval $\Delta t'$. Therefore, the force of the striker (F_s) is

$$F_s = m_s \frac{dv_s}{dt} = (\rho_s A_s L') \left(\frac{V - U_p}{L' / C_s} \right) = A_s \rho_s C_s (V - U_p) \quad (10)$$

and the striker stress (σ_s) is

$$\sigma_s = \frac{F_s}{A_s} = \rho_s C_s (V - U_p) \quad (11)$$

Note that in Figure 2(b) and equation (11), if the particle velocity at the front-side of the striker (U_p) moves at the same velocity as V , no stress builds up in the striker; the stress in the striker (σ_s) builds up because of the difference between V and U_p . The decrease in the amount of the particle velocity ($V - U_p$) is the driving force for the stress build-up in the striker: (impact-generated striker stress) = (striker impedance) $\times$ (decrease amount of particle velocity). Except for the impact-generated striker stress, the stress of a medium (striker or bar) is given by the change in the amount of the particle velocity (ΔU_p): (medium stress) = (medium impedance) $\times$ ΔU_p . For instance, in the case of the bar stress (σ_b) in Figure 2(b), ΔU_p equals U_p because the particle velocity was zero before the impact. As such, the latter relationship (change amount relationship) leads to equation (9) for the bar stress. The above two relationships are significant for the release stress of the striker, which will be described in the 'Release wave' section.

Equations (9) and (11) describe the bar stress (σ_b) and striker stress (σ_s), respectively, in terms of the particle velocity (U_p). For practical use of these equations, it is desirable to express the particle velocity (U_p) and stresses (σ_s and σ_b) in terms of the impact velocity (V) like equations (6) and (7). In this regard, momentum conservation is considered at the moment ($\Delta t = L/C_s$) when the travelling wave in the striker reaches the rear end of the striker, which is shown in Figure 2(c). At the moment of Δt (Figure 2(c)), the travel distance of the wave in the bar is $\Delta t C_b = L C_b / C_s$. From the consideration that the momenta in the striker and bar at $\Delta t = L/C_s$ are the same as the momentum in the striker before the impact

$$\rho_s A_s L V = \rho_s A_s L U_p + \rho_b A_b (L C_b / C_s) U_p \quad (12)$$

where the left side is the momentum of the striker before the impact ($t \leq 0$) and the right side is the

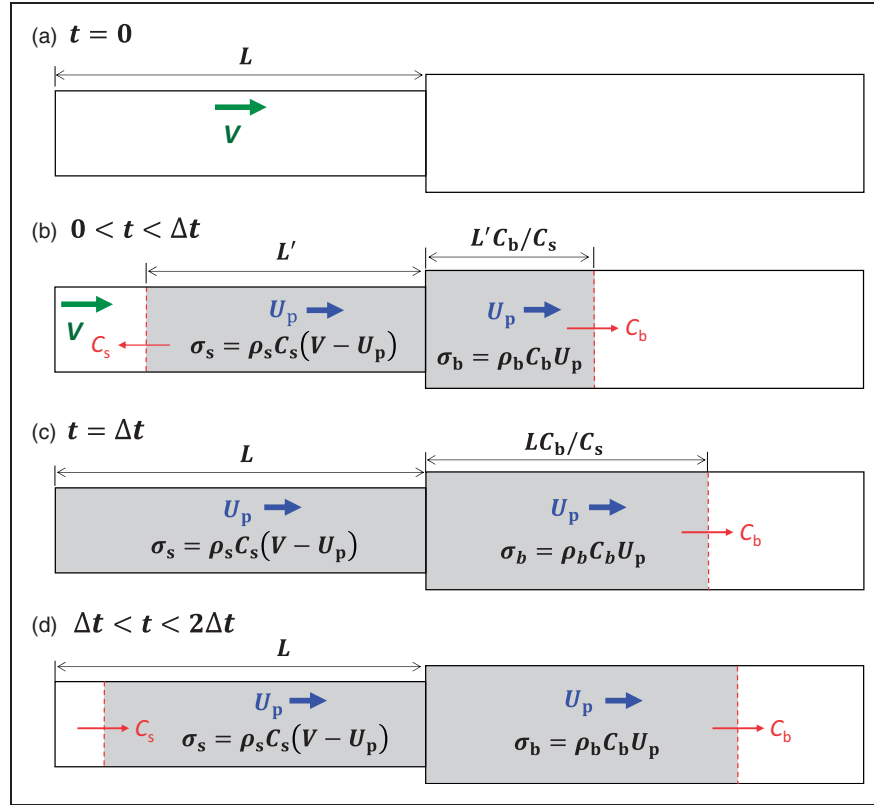


Figure 2. Stress build up and particle velocity around a general impact surface across which either cross-sectional areas or materials are different. L is length of striker.

momentum of the striker (first term) plus that of the bar (second term) when $\Delta t = L/C_s$ (Figure 2(c)). Equation (12) can be transformed to

$$U_p = \frac{Z_s}{Z_s + Z_b} V \quad (13)$$

where $Z = A\rho C$. Equation (13) correlates the particle velocity with the impact velocity. It reduces to equation (6) if $Z_s = Z_b$.

The bar stress (Figure 2) after the impact on the general impact surface can be obtained by combining equations (9) and (13):

$$\sigma_b = \rho_b C_b U_p = \frac{Z_s}{Z_s + Z_b} \rho_b C_b V \quad (14)$$

Similarly, the striker stress (Figure 2) can be obtained by combining equations (11) and (13):

$$\sigma_s = \rho_s C_s (V - U_p) = \frac{Z_b}{Z_s + Z_b} \rho_s C_s V \quad (15)$$

Equations (14) and (15) describe the bar stress (σ_b) and striker stress (σ_s), respectively, in terms of impact velocity (V). When $Z_s = Z_b$, equations (14) and (15) are reduced to equation (7).

The impact-generated particle velocity (U_p), bar stress (σ_b) and striker stress (σ_s) were calculated using equations (13) to (15) for a range of Z_s/Z_b

ratios using the properties and geometries listed in Table 1. The impact velocity (V) was 15 m/s. The result is listed in Table 2. The release particle velocity (U_r) and release stress (σ_r) in Table 2 will be explained later in the 'Release wave' section.

Release wave

Impact on rigid surface

To investigate the phenomenon when the impact-generated stress wave reaches the rear end of the striker, the case where the striker impacts on a rigid wall is first considered, as shown in Figure 3. The striker moves initially at speed V . Upon impact (Figure 3(a)), the striker particles stop sequentially from the front surface of the striker. Information on particle stoppage propagates towards the rear surface at speed C_s . The particle velocity and stress at a moment before the information of the particle stoppage reaches the rear end of the striker is shown in Figure 3(b), where mark '0' indicates the region with a zero-particle velocity. Each particle in this region experienced a velocity decrease amount of ' $V - 0$ ' when particle stoppage information arrived, and thus the stress in this region (σ_s) is increased by $\rho_s C_s V$.

At $t = \Delta t$ (Figure 3(c)), the stress wave (information of particle stoppage) reaches the rear end of the striker. As the velocity of all particles in the striker is zero, the whole striker is momentarily stationary.

Table 2. Calculated values of particle velocity (U_p), striker stress (σ_s), bar stress (σ_b), release particle velocity (U_r) and release stress (σ_r) using the properties and geometries listed in Table 1 for a range of Z_s/Z_b ratios. Impact velocity (V) was 15 m/s.

Z_s/Z_b	U_p (m/s) $= \frac{Z_s}{Z_s+Z_b} V$	σ_s $= \rho_s C_s (V - U_p)$ $= \frac{Z_b}{Z_s+Z_b} \rho_s C_s V$	σ_b $= \rho_b C_b U_p$ $= \frac{Z_s}{Z_s+Z_b} \rho_b C_b V$	U_r (m/s) $= -(V - 2U_p)$ $= \frac{Z_s-Z_b}{Z_s+Z_b} V$	σ_r $= -\rho_s C_s (V - U_p)$ $= -\frac{Z_b}{Z_s+Z_b} \rho_s C_s V$
1/4	3.0	$12\rho_s C_s$	$3\rho_b C_b$	-9.0	$-12\rho_s C_s$
1/2	5.0	$10\rho_s C_s$	$5\rho_b C_b$	-5.0	$-10\rho_s C_s$
1	7.5	$7.5\rho_s C_s$	$7.5\rho_b C_b$	0.0	$-7.5\rho_s C_s$
2	10.0	$5\rho_s C_s$	$10\rho_b C_b$	5.0	$-5\rho_s C_s$
4	12.0	$3\rho_s C_s$	$12\rho_b C_b$	9.0	$-3\rho_s C_s$

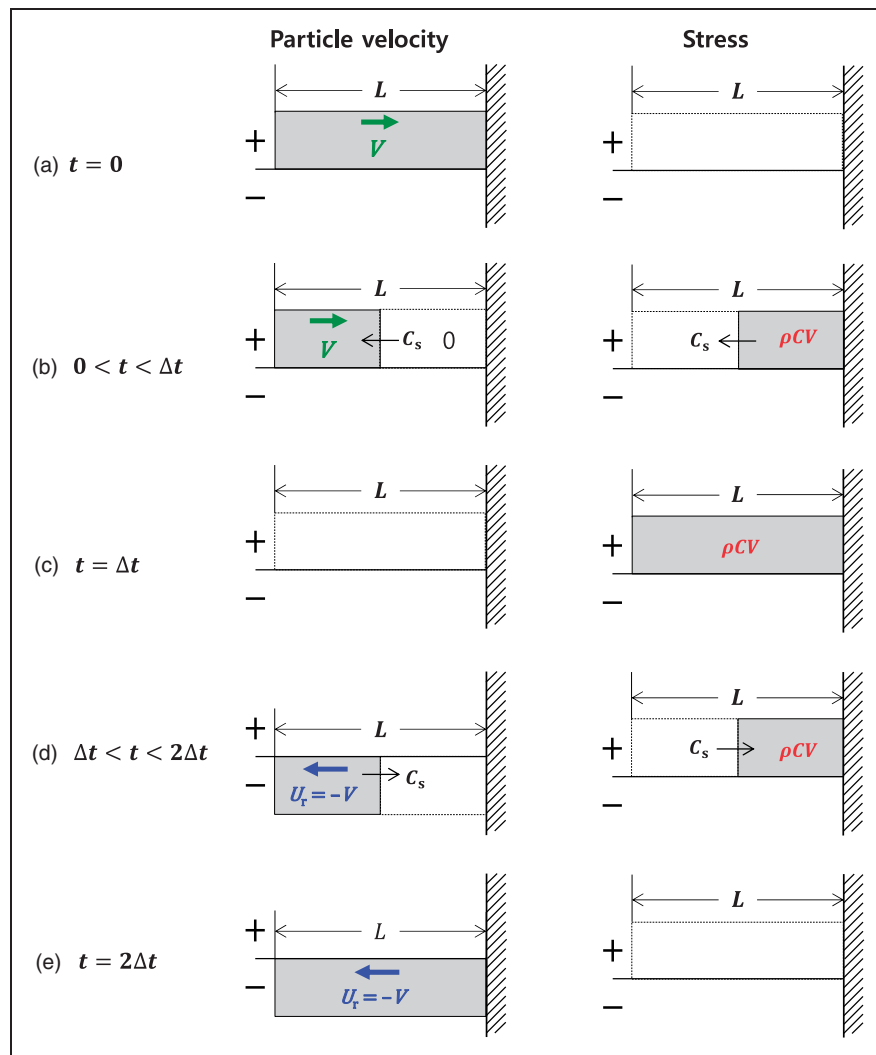


Figure 3. Particle velocity and stress build-up for the case where striker impacts on rigid wall. L is length of striker.

The axial distance between the particles is minimized throughout the striker. At this moment, the only way to *relieve* the compressed state of the striker particles is their movement to the left (negative) direction. This *release* motion starts from the rear end of the striker, which information propagates towards the front of the striker at speed C_s (Figure 3(d)). Assuming the

purely elastic behaviour of the striker particles, the *release* particle velocity (U_r) is

$$U_r = -V \quad (16)$$

As the release motion occurs at $\Delta t \leq t \leq 2\Delta t$, the striker particles experience velocity change from zero

($t = \Delta t$) to $-V$ ($\Delta t < t \leq 2\Delta t$). Because the *change amount* of the particle velocity is $-V$, the stress associated with the release particle motion from $t = \Delta t$, i.e. the release stress, σ_r , is

$$\sigma_r = -\rho_s C_s V \quad (17)$$

This *tensile* release stress generated from the rear side of the striker completely relieves (nullifies) the initially compressed state ($\sigma_s = \rho_s C_s V$) of the striker particles due to impact.

In Figure 3(e), when the release wave with a particle velocity of $-V$ reaches the front of the striker, the compressive stress in the whole striker volume is completely relieved. Then, the whole striker translates backwards in accordance with the negative particle velocity of the release wave ($U_r = -V$). Therefore, the striker is separated from the rigid wall at $t > 2\Delta t$. The momentum of the striker at $t = 0$ is conserved at $t = 2\Delta t$. No momentum is transferred to the rigid wall; the whole magnitude of the initial momentum is used for the backwards translation of the striker after the impact.

The phenomena of the stress build-up and its release in the case of the impact on a rigid wall can be described in general terms as follows. In Figure 3(b), the driving force to build up the striker stress, i.e. the velocity *decrease amount* is $V - U_p$, where U_p is zero in the case of the impact on the rigid wall. As a result, the magnitude of the built-up stress is $\rho_s C_s (V - U_p) = \rho_s C_s V$. Because this amount of the stress has to be relieved, the release stress is $-\rho_s C_s (V - U_p) = -\rho_s C_s V$, and thus, the release particle velocity is $-(V - U_p) = -V$. In the case of impact on the rigid wall, the stress rise in the striker is maximal ($\sigma_s = \rho_s C_s (V - U_p) = \rho_s C_s V$), and therefore, the maximum release particle velocity is achieved as $U_r = -V$ (equation (16)). The general impact surface where U_p is non-zero is considered in the next sections.

For the case of a rigid impact surface, at $t = \Delta t$ (Figure 3(c)), it can be considered that there is a hypothetical exit located at the left side of the stressed zone. The axial position of the exit is fixed in Eulerian coordinates at the rear end of the striker. Then, it can be considered that the compressed striker particles escape through the exit at a velocity of $-V$ to relieve their compressed state.

Impact on higher-general-impedance surface ($Z_s < Z_b$)

Consider now the case where $Z_s < Z_b$, in which the magnitude of Z_b is finite. The impact-generated particle velocity and stress in this case are shown in Figure 4. As can be observed in Figure 4(a), the driving force to build up the striker stress, i.e. the velocity decrease amount, is $V - U_p$, where U_p is non-zero. As a result, the magnitude of the built-up stress (σ_s) is $\rho_s C_s (V - U_p)$, which is smaller than that for the

impact on a rigid surface ($\rho_s C_s V$). This magnitude of built-up stress, $\rho_s C_s (V - U_p)$, has to be relieved at $\Delta t \leq t \leq 2\Delta t$.

If the impact interface were stationary as in the case of impact on a rigid wall (if there were only one exit on the left side of the stressed zone), the release particle velocity (U_r) would have been $-(V - U_p)$, because the striker stress that must be relieved is only $\rho_s C_s (V - U_p)$. However, it is important to note that, when the stress wave (information on particle deceleration) reaches the rear end of the striker at $t = \Delta t$ (Figure 4(b)), the entirety of the striker particles are already moving in the right direction at U_p , unlike the case of the impact on the rigid wall (Figure 3(c)). This situation is as if there were another exit on the right side of the stressed zone at the front of the striker (impact surface); a percentage of the compressed state is self-relieved via particle movement in the right direction through the right exit (impact surface) at speed U_p . Therefore, the magnitude of particle velocity ($|U_r|$) starting from the rear end of the striker decreases from $|V - U_p|$ by the amount of the right-moving particle velocity ($|U_p|$). The net magnitude of the release particle velocity ($|U_r|$) is then $|V - U_p|$ minus the right-moving particle velocity, $|U_p|$

$$U_r = -(V - 2U_p) \quad (18)$$

As the release motion occurs at $\Delta t \leq t \leq 2\Delta t$, the velocity of striker particles changes from U_p at $t = \Delta t$ to $U_r = -(V - 2U_p)$ at $\Delta t < t \leq 2\Delta t$. Therefore, the change amount of the particle velocity is $\Delta U_p = U_r - U_p = -(V - U_p)$. Then, the stress of the release wave, σ_r , is

$$\sigma_r = \rho_s C_s \Delta U_p = \rho_s C_s (U_r - U_p) = -\rho_s C_s (V - U_p) \quad (19)$$

This *tensile* release stress completely relieves (nullifies) the initially compressed state ($\sigma_s = \rho_s C_s (V - U_p)$) of the striker particles due to the impact. As the release wave reaches the front of the striker at $t = 2\Delta t$ (Figure 4(d)), the whole striker volume is stress-free.

The magnitude of the release particle velocity (U_r) and release stress (σ_r) can be calculated using information on impact velocity (V). If equation (13) ($U_p = Z_s / (Z_s + Z_b) \cdot V$) is applied to equation (18), the latter transforms to

$$U_r = \frac{Z_s - Z_b}{Z_s + Z_b} V \quad (20)$$

Application of equation (13) to equation (19) leads to

$$\sigma_r = -\frac{Z_b}{Z_s + Z_b} \rho_s C_s V \quad (21)$$

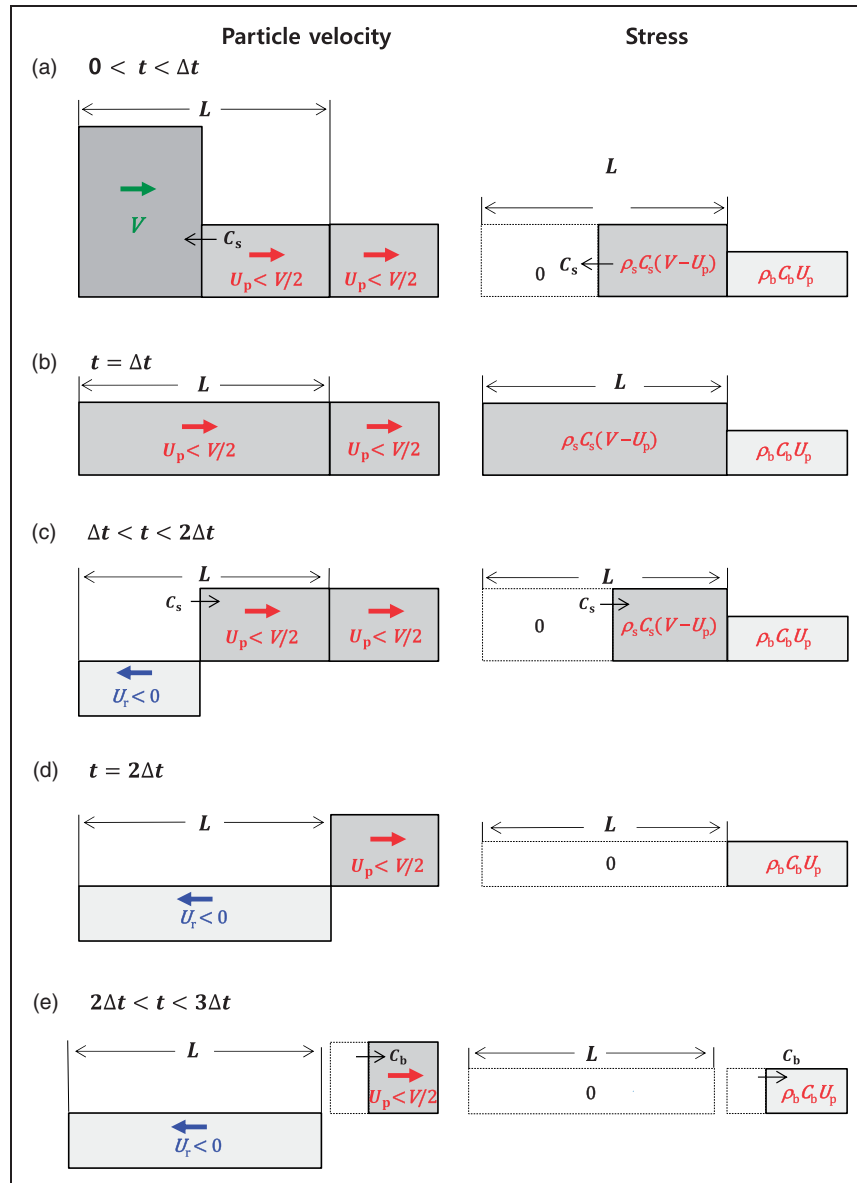


Figure 4. Particle velocity and stress build-up for the case where $Z_s < Z_b$. L is length of striker. Magnitude of U_r can be either smaller or larger than that of U_p (see equations (18), (20) and Table 2).

The calculated values of the release particle velocity (U_r) and release stress (σ_r) using equations (20) and (21), respectively, are listed in Table 2 for a range of Z_s/Z_b ratios at an example impact velocity of 15 m/s.

According to equation (13) ($U_p = Z_s / (Z_s + Z_b) \cdot V$), the interface velocity U_p is $V/2$ when $Z_s = Z_b$ while the value is smaller than $V/2$ if $Z_s < Z_b$. Therefore, the sign of the release particle velocity (U_r) in equations (18) and (20) remains negative (left-travelling). In this regard, U_r in Figure 4(c) is drawn to be negative; the striker translates backwards at $t > 2\Delta t$. This phenomenon is qualitatively the same as the case of the impact on a rigid surface.

On a quantitative base, however, the speed of the backwards translation in the considered case ($Z_s < Z_b$) is lower than the case of the rigid surface ($U_r = -V$) according to equations (18) and (20). If the case where $Z_s/Z_b = 0.5$ is considered, the backwards

translation velocity (U_r at $t = 2\Delta t$) is $-V/3$. This result means that only two-third of the striker momentum (mV) is transferred to the bar, and the remaining one-third of the striker momentum is used for the backwards translation of the striker after impact ($t > 2\Delta t$). Therefore, the momentum transfer is inefficient in the case where $Z_s < Z_b$.

Impact on same-general-impedance surface ($Z_s = Z_b$)

The impact-generated particle velocity and stress in the case where $Z_s = Z_b$ are illustrated in Figure 5. As mentioned, the particle velocity (equation (13)) and striker stress (equation (15)) in this case are $V/2$ and $\rho CV/2$, respectively. This magnitude of built-up stress, $\rho CV/2$, has to be relieved at $\Delta t \leq t \leq 2\Delta t$.

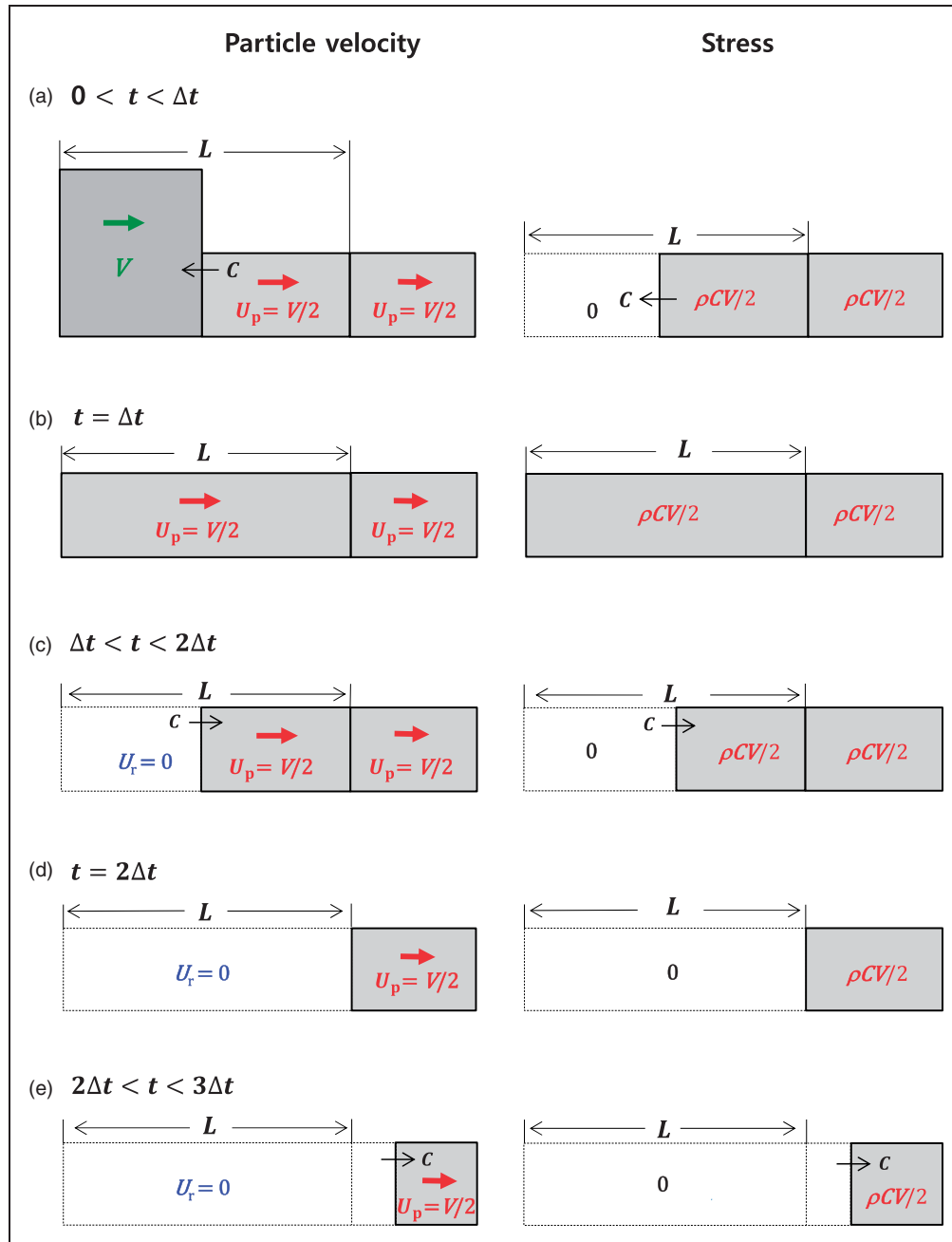


Figure 5. Particle velocity and stress build-up for the case where $Z_s = Z_b$. L is length of striker. $C = C_s = C_b$.

In the previous case of a rigid wall (Figure 3) where striker particles were stationary at $t = \Delta t$, a maximum magnitude of the release particle velocity moving in the left (negative) direction was necessary ($U_r = -V$) to relieve the striker stress (ρCV) because no particles were self-relieving the compressed state by moving in the right (as opposed to left) direction. In the case where $Z_s < Z_b$ (Figure 4), the compressed state of the striker at $t = \Delta t$ was being self-relieved to a certain extent because the striker particles were moving to the right (positive) direction at a speed of $U_p < V/2$. Therefore, a diminished magnitude of the release particle velocity moving to the left direction was necessary at the rear end of the striker: $U_r = -(V - 2U_p) < 0$. In the considered case, where

$Z_s = Z_b$ (Figure 5), the self-release motion of the striker particles to the right direction is sufficiently enhanced in the stressed region so as to make the U_p value be $V/2$, by which the compressed state of the striker particles ($\sigma_s = \rho CV/2$) is completely relieved. Therefore, there is no reason for the striker particles to move in the left (negative) direction at the rear end of the striker: $U_r = 0$. This magnitude of U_r is readily obtained using equations (18) and (20) when $Z_s = Z_b$.

At $\Delta t \leq t \leq 2\Delta t$, the velocity of striker particles changes from $U_p = V/2$ at $t = \Delta t$ to $U_r = -(V - 2U_p) = 0$ at $\Delta t < t \leq 2\Delta t$. Therefore, the *change amount* of the particle velocity is: $\Delta U_p = U_r - U_p = -V/2$. Then, the release stress is

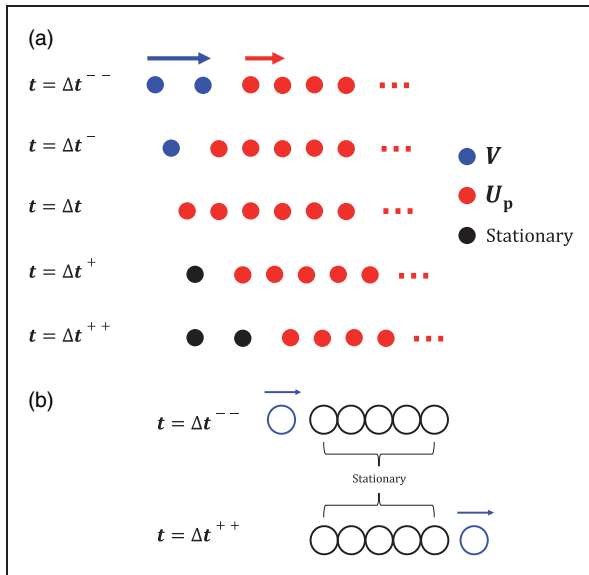


Figure 6. (a) Particle stoppage starting from rear end of striker at $t = \Delta t$ and (b) pendulum experiment where impacting pendulum on left is still after transferring its momentum to right pendulums.

$\sigma_r = (U_r - U_p)\rho_s C_s = -\rho_s C_s V/2$ (equations (19) and (21)). This tensile release stress completely eliminates the impact-generated striker stress, $\sigma_s = \rho C V/2$.

Figure 6 depicts the particle motion that makes the release particle velocity zero when $Z_s = Z_b$. In Figure 6(a), $t = \Delta t$ is the moment when the end particles of the striker impact the second-layer particles beneath the end surface of the striker. At $t \geq \Delta t$, the end surface particles stop because the momentum of the end particles was already transferred to the particles on the right side. This phenomenon has an analogy to the well-known pendulum impact (see Figure 6(b)), where the material and diameter of the pendulums are the same like the case where $Z_s = Z_b$. In Figure 6(b), the impacting pendulum in the left side is still after transferring its momentum to the right pendulum. Likewise, in Figure 6(a), after the last particles at the rear end of the striker stop, the stationary particle region grows towards the right side because the momentum transfer to the right-sided particles is completed.

Note in Figure 5(a) that, when $Z_s = Z_b$, the impact-generated compressive stress waves with the same magnitude propagate towards the free (rear) surfaces of the *striker* as well as *bar*. It is well known that a compressive stress wave in the *bar* returns from the free end surface as a tensile wave.³ However, there is no reflected tensile wave from the rear (free) end of the *striker*. The particle stoppage and growth of the stationary region from the rear end of the striker (Figure 6) explain why there is no reflected tensile wave from the rear end of the *striker*.

When the information on particle stoppage reaches the front of the striker, i.e. when the pulse time is over

(Figure 5(d)), the striker is stationary ($U_r = 0$). Therefore, in the case, where $Z_s = Z_b$, a 100% of the striker momentum is transferred to the bar; this case is the most efficient in transferring the striker momentum to the bar.

Impact on lower-general-impedance surface ($Z_s > Z_b$)

Consider now the case where $Z_s > Z_b$. The impact-generated particle velocity and stress for this case are illustrated in Figure 7. For convenience, the magnitudes of the particle velocity and stress in Figure 7 are explained below for a specific Z_s/Z_b ratio of 2 and an impact velocity (V) of 15 m/s. Figure 7(a) ($0 < t < \Delta t$) depicts the build-up of the particle velocity and stress in accordance with the previous discussions. For instance, the particle velocity (U_p) of the impact-generated primary wave is 10 m/s according to equation (13) ($U_p = Z_s/(Z_s + Z_b) \cdot V = 2V/3$). As a result, $\sigma_s = \rho_s C_s (V - U_p) = 5\rho_s C_s$ and $\sigma_b = \rho_b C_b U_p = 10\rho_b C_b$ (equations (11) and (9), respectively). At $t = \Delta t$ (Figure 7(b)), this impact information on stress and particle velocity reaches the rear end of the striker.

Judging from the magnitude of the particle velocity, the case where $Z_s/Z_b = 1$ with a U_p value of $V/2$ lies right in the middle between the rigid surface ($U_p = 0$; $Z_s/Z_b = 0$) and free surface ($U_p = V$; $Z_s/Z_b = \infty$). Therefore, the impedance of the bar in the considered case ($Z_s/Z_b > 1$; $U_p > V/2$) is now closer to the free surface ($Z_s/Z_b = \infty$; $U_p = V$) than to the rigid surface ($Z_s/Z_b = 0$; $U_p = 0$). Because $U_p > V/2$, the release particle velocity ($U_r = -(V - 2U_p)$ in equation (18)) is now positive; particles move to the right (positive) direction. In the considered case of a Z_s/Z_b value of 2 and impact velocity of 15 m/s, equation (20) ($U_r = (Z_s - Z_b)/(Z_s + Z_b) \cdot V$) results in the release particle velocity (U_r) as +5 m/s. In this regard, U_r is drawn to be positive in Figure 7(c).

As the release motion occurs at $\Delta t \leq t \leq 2\Delta t$, the striker particles experience velocity change from $U_p = +10$ m/s at $t = \Delta t$ to $U_r = -(V - 2U_p) = +5$ m/s at $\Delta t < t \leq 2\Delta t$; the *change amount* of the particle velocity is $\Delta U_p = (U_r - U_p) = -(V - U_p) = -5$ m/s. Therefore, or according to equation (19), the stress of the release wave, σ_r , is: $-\rho_s C_s (V - U_p) = -5 \rho_s C_s$.

This *tensile* release stress completely relieves (nullifies) the initially compressed state ($\sigma_s = \rho_s C_s (V - U_p) = +5\rho_s C_s$) of the particles generated by impact. Therefore, the stress in the release wave region in Figure 7(c) ($\Delta t < t < 2\Delta t$) is drawn to be stress-free (marked as zero).

As the release wave reaches the front of the striker at $t = 2\Delta t$ (Figure 7(d)), the whole striker volume is *stress-free* and *forwards translating* at +5 m/s ($= U_r$). The striker at this moment is a free-flying object like the initial striker that impacts the bar at $t = 0$; the only

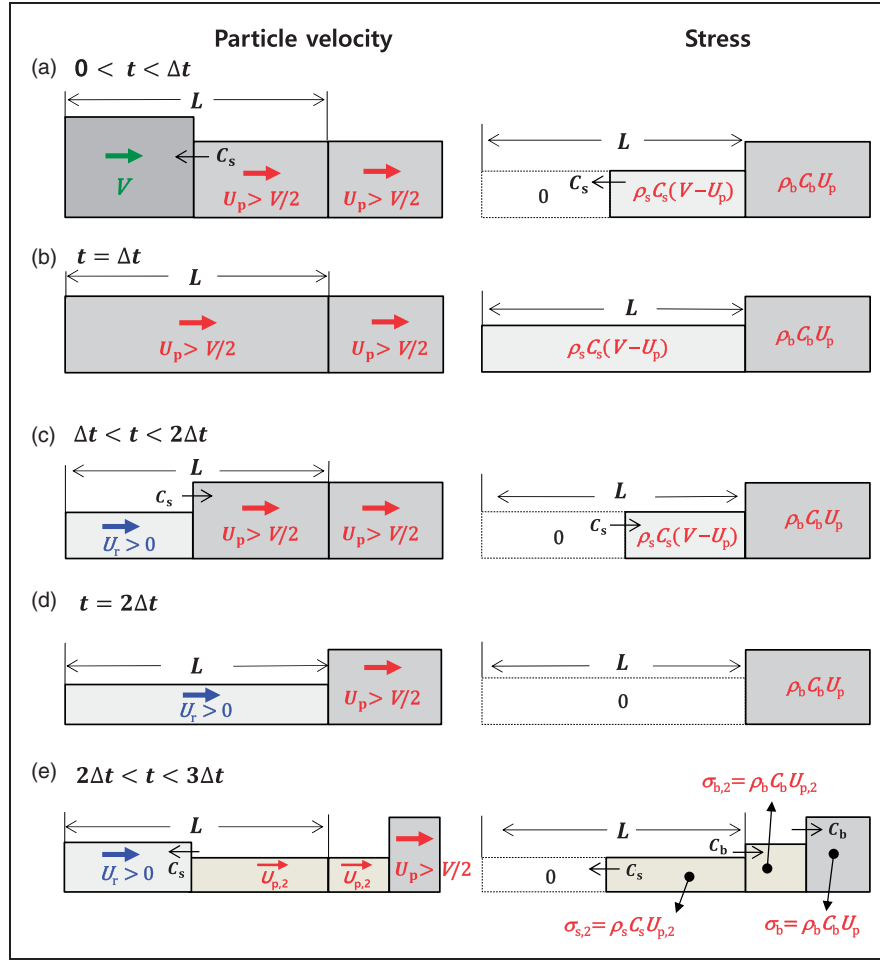


Figure 7. Particle velocity and stress build-up for the case where $Z_s > Z_b$. L is length of striker. Magnitude of U_r is smaller than U_p (see equations (18), (20) and Table 2).

difference being that the flight velocity is now only +5 m/s and impacts the bar at $t = 2\Delta t$. At this moment, a new impact (the second impact) cycle starts, and the first impact cycle is completed.

At $t = 2\Delta t$, the striker impacts the end particles of the bar that would have been slowed down to the zero or negative velocity if there were no second impact cycle like the cases where $Z_s = Z_b$ and $Z_s < Z_b$, respectively. The truth when $Z_s > Z_b$ is that there is a positive release particle velocity (U_r) of the first cycle that serves as the impact velocity for the second cycle ($V_2 = U_{r,1} = U_r = +5$ m/s). Subscripts 1 and 2 here denote the first and second impact cycles.

The new particle velocity ($U_{p,2}$) in the striker and bar in the second impact cycle ($2\Delta t < t < 3\Delta t$; Figure 7(e)) can be calculated using equation (13) ($U_{p,2} = Z_s / (Z_s + Z_b) \cdot V_2$), and V_2 is now +5 m/s (the U_r value in the first impact cycle). For the considered case, where $Z_s/Z_b = 2$ and the initial impact velocity (V) is 15 m/s, the new particle velocity ($U_{p,2}$) in the second cycle of impact is calculated to be $2V_2/3$ ($= 10/3 = 3.33$ m/s). This magnitude of the particle velocity is schematically illustrated at both sides of the impact surfaces in Figure 7(e) ($2\Delta t < t < 3\Delta t$).

Because the new particle velocity $U_{p,2}$ is +3.33 m/s, the new bar stress ($\sigma_{b,2} = \rho_b C_b U_{p,2}$) is calculated to be $3.33\rho_b C_b$. This value can also be calculated using equation (14) ($\sigma_{b,2} = Z_s / (Z_s + Z_b) \cdot \rho_b C_b V_2$), where V_2 (U_r in the previous cycle) is +5 m/s. The new striker stress ($\sigma_{s,2}$) is calculated to be $1.67\rho_s C_s$ using equation (11), $\sigma_{s,2} = \rho_s C_s (V_2 - U_{p,2})$, or equation (15), $\sigma_{s,2} = Z_b / (Z_s + Z_b) \cdot \rho_s C_s V_2$.

In Figure 7(d) ($t = 2\Delta t$), the flight (translation) velocity of the striker is $V/3$ ($= +5$ m/s) according to equation (20) ($U_r = (Z_s - Z_b) / (Z_s + Z_b) \cdot V$). For the considered case, where the Z_s/Z_b ratio is 2, a one-third of the striker momentum still remains in the striker at $t = 2\Delta t$, which means that only a two-third of the striker momentum is transferred by the time $t = 2\Delta t$. Recall that the same ratio (2/3) of the momentum was transferred in the case where $Z_s/Z_b = 0.5$. The difference between the two cases ($Z_s/Z_b = 0.5$ and 2.0) is that, in the case where the Z_s/Z_b ratio was 0.5, the one-third of the original momentum left in the striker was used for the backwards translation of the striker, while, in the current case, where the Z_s/Z_b ratio is 2.0, the one-third of the original momentum left in the striker is transferred to

the bar at later impact cycles as will be described in the next section ('Multiple impact cycles when $Z_s > Z_b$ ' section).

Multiple impact cycles when $Z_s > Z_b$

The start of the impact-generated stress from the front to the rear end of the striker and the arrival of the release wave from the rear to the front of the striker form one cycle of impact. As was seen in Figure 7, once the release wave reaches the front of the striker, the stress and particle velocity in the whole striker reset themselves: the new striker stress is zero, and the new impact velocity is the release particle velocity in the first cycle: $U_r = (Z_s - Z_b)/(Z_s + Z_b) \cdot V$. At $t = 2\Delta t$, the stress-free striker imposes a new impact (a new cycle) on the bar end, which is moving at the particle velocity of the previous cycle.

In this way, multiple impact cycles take place when $Z_s > Z_b$. Each impact cycle is completed at every $2\Delta t$ interval. These multiple impact cycles are illustrated in Figure 8, where subscripts, 1, 2, 3, ... and k denote the cycle number; and V_k is the impact velocity that triggers the k th cycle. As can be observed, the release particle velocity of the previous ($k - 1$)th cycle ($U_{r,k-1}$) serves as the impact velocity (V_k) in the next (k)th cycle.

From equation (20), the release particle velocity of the first cycle ($U_{r,1}$) is given by $U_r = (Z_s - Z_b)/(Z_s + Z_b) \cdot V$ (equation (20)). The release particle velocity of the second cycle is

$$U_{r,2} = \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right) V_2 = \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right) U_{r,1} = \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^2 V \quad (22)$$

In this way, the release particle velocity of the k th cycle ($U_{r,k}$) is

$$U_{r,k} = \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^k V \quad (23)$$

In equation (23), $U_{r,1}$ when $k=1$ is the release particle velocity of the first cycle. Because the impact velocity of the k th cycle, V_k , is the release particle velocity of the previous ($k - 1$)th cycle, it follows that

$$V_k = U_{r,k-1} = \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} V \quad (24)$$

In the frame work of equation (24), $U_{r,0}$ is resulted when $k=1$ (the first cycle). This $U_{r,0}$ literally indicates the release particle velocity of the zeroth cycle but there is actually no zeroth impact cycle; $U_{r,0}$ is a hypothetical term, which magnitude has the same value as the impact velocity in the first cycle, V_1 .

The particle velocity (U_p) in the first cycle is given by $U_p = Z_s/(Z_s + Z_b) \cdot V$ (equation (13)). The particle velocity of the second cycle is

$$\begin{aligned} U_{p,2} &= \left(\frac{Z_s}{Z_s + Z_b}\right) V_2 = \left(\frac{Z_s}{Z_s + Z_b}\right) U_{r,1} \\ &= \left(\frac{Z_s}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right) V \end{aligned} \quad (25)$$

In this way, the particle velocity of the k th cycle ($U_{p,k}$) is

$$U_{p,k} = \left(\frac{Z_s}{Z_s + Z_b}\right) V_k = \left(\frac{Z_s}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} V \quad (26)$$

The striker stress in the first cycle is given by $\sigma_s = Z_b/(Z_s + Z_b) \cdot \rho_s C_s V$ (equation (15)). The striker stress in the second cycle is

$$\begin{aligned} \sigma_{s,2} &= \left(\frac{Z_b}{Z_s + Z_b}\right) \rho_s C_s V_2 = \left(\frac{Z_b}{Z_s + Z_b}\right) \rho_s C_s U_{r,1} \\ &= \left(\frac{Z_b}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right) \rho_s C_s V \end{aligned} \quad (27)$$

In this way, the striker stress in the k th cycle is

$$\begin{aligned} \sigma_{s,k} &= \left(\frac{Z_b}{Z_s + Z_b}\right) \rho_s C_s V_k \\ &= \left(\frac{Z_b}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} \rho_s C_s V \end{aligned} \quad (28)$$

Release stress ($\sigma_{r,k}$) occurs at $k\Delta t \leq t \leq 2k\Delta t$. It always relieves the impact-generated striker stress ($\sigma_{s,k}$): $\sigma_{r,k} = -\sigma_{s,k}$. Therefore, the release stress in the k th cycle is

$$\begin{aligned} \sigma_{r,k} &= -\left(\frac{Z_b}{Z_s + Z_b}\right) \rho_s C_s V_k \\ &= -\left(\frac{Z_b}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} \rho_s C_s V \end{aligned} \quad (29)$$

The bar stress in the first cycle is given by $\sigma_b = Z_s/(Z_s + Z_b) \cdot \rho_b C_b V$ (equation (14)). The bar stress in the second cycle is

$$\begin{aligned} \sigma_{b,2} &= \left(\frac{Z_s}{Z_s + Z_b}\right) \rho_b C_b V_2 = \left(\frac{Z_s}{Z_s + Z_b}\right) \rho_b C_b U_{r,1} \\ &= \left(\frac{Z_s}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right) \rho_b C_b V \end{aligned} \quad (30)$$

In this way, the bar stress in the k th cycle is

$$\begin{aligned} \sigma_{b,k} &= \left(\frac{Z_s}{Z_s + Z_b}\right) \rho_b C_b V_k \\ &= \left(\frac{Z_s}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} \rho_b C_b V \end{aligned} \quad (31)$$

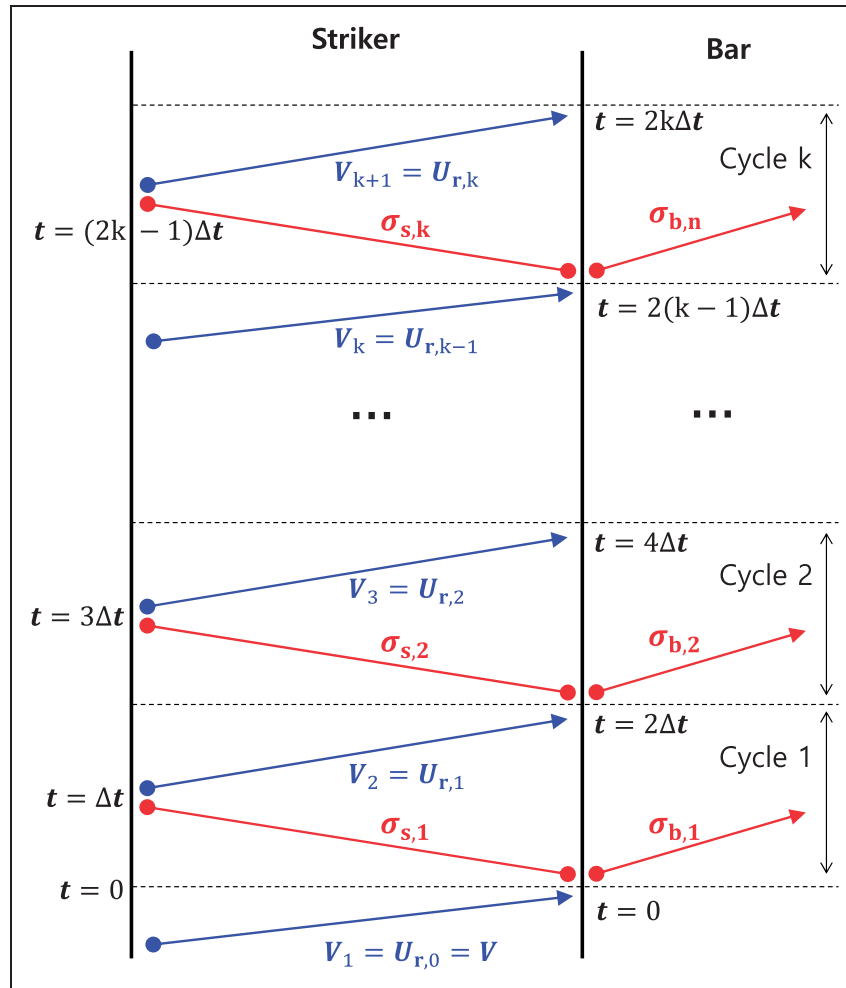


Figure 8. Multiple impact cycles in striker and resultant waves in bar. Closed circles indicate starting/ending points of respective waves. k denotes number of cycles and $U_{r,0}$, the release particle velocity of the zeroth cycle, is only a hypothetical term that simply indicates the impact velocity of the first cycle, V_1 .

For a special case, where the cross-sectional areas of the striker and bar are identical with only different impedances (ρC), equations (28) and (31) are reduced to the same equation:

$$\sigma_{s,k} = \sigma_{b,k} = \left(\frac{\rho_s C_s \rho_b C_b}{\rho_s C_s + \rho_b C_b} \right) \left(\frac{\rho_s C_s - \rho_b C_b}{\rho_s C_s + \rho_b C_b} \right)^{k-1} V \quad (32)$$

If the cross-sectional areas of the striker and bar are different for the same striker/bar material, equations (28) and (31) will be necessary instead of equation (32) for calculating stresses in the striker and bar, respectively.

1D prediction and numerical verification

1D prediction

The particle velocity (U_p) in the striker and bar was calculated using the 1D theory formulated in the previous sections. Varying Z_s/Z_b ratios were

considered for the properties and geometries of the striker and bar listed in Table 1. The result is presented in Figure 9, where the time was set for the particles at the impact surfaces of the striker and bar.

For the cases where $Z_s/Z_b = 0.5$ and 1.0 , the magnitude of the particle velocity (U_p) during the first impact cycle ($0 \leq t \leq 2\Delta t$) was calculated using equation (13) ($U_p = Z_s/(Z_s + Z_b) \cdot V$). In these cases, the striker is separated or stationary at $t \geq 2\Delta t$ (Figures 4 and 5, respectively). Because the U_p value of the striker after the primary pulse is the U_r value in the primary pulse period, the U_p value at $t \geq 2\Delta t$ was calculated using equation (20) ($U_r = (Z_s - Z_b)/(Z_s + Z_b) \cdot V$). The calculated striker particle velocities were -5 and 0 m/s for the cases of $Z_s/Z_b = 0.5$ and 1.0 , respectively, which values are drawn in Figure 9(a) and (b), respectively. For the case where $Z_s/Z_b = 2$, the U_p value of the striker was calculated using equation (26) ($U_{p,k} = \left(\frac{Z_s}{Z_s + Z_b} \right) \left(\frac{Z_s - Z_b}{Z_s + Z_b} \right)^{k-1} V$). The calculated results are drawn in Figure 9(c), which show a series of step-wise residual pulses.

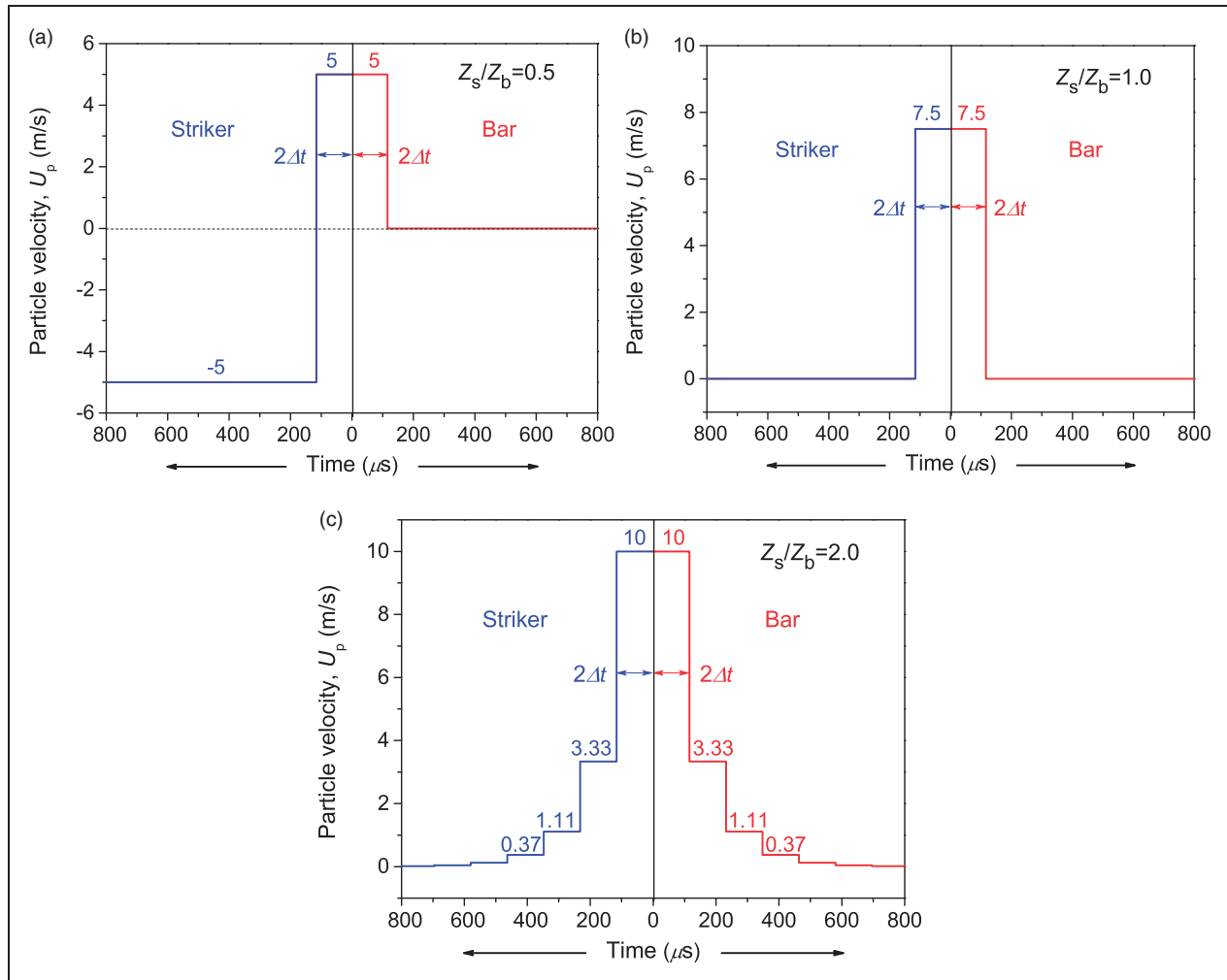


Figure 9. Particle velocity (U_p) at the impact surfaces of striker and bar. Δt is one-way travel time of elastic wave in striker. Impact velocity was 15 m/s and Z_s/Z_b ratios were: (a) 0.5, (b) 1.0 and (c) 2.0.

The stresses in the striker (σ_s) and bar (σ_b) were also calculated using the equations formulated in this study. For the cases where $Z_s \leq Z_b$, σ_b and σ_s were calculated using equations (14) ($\sigma_b = \frac{Z_s}{Z_s + Z_b} \rho_b C_b V$) and (15) ($\sigma_s = \frac{Z_b}{Z_s + Z_b} \rho_s C_s V$), respectively. For the case where $Z_s > Z_b$, $\sigma_{s,k}$ and $\sigma_{b,k}$ at multiple impact cycles were calculated using equations (28) ($\sigma_{s,k} = \left(\frac{Z_b}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} \rho_s C_s V$) and (31) ($\sigma_{b,k} = \left(\frac{Z_s}{Z_s + Z_b}\right) \left(\frac{Z_s - Z_b}{Z_s + Z_b}\right)^{k-1} \rho_b C_b V$), respectively. The results are presented in Figure 10.

As can be observed in Figure 10, unlike the case of the particle velocity (Figure 9), the values of σ_s and σ_b are the same only when the Z_s/Z_b ratio is unity. The primary *striker* stress decreases (406.02 $\rightarrow$ 304.51 $\rightarrow$ 203.01 MPa) with the Z_s/Z_b ratio while the primary *bar* stress increases (203.01 $\rightarrow$ 304.51 $\rightarrow$ 406.02 MPa), as they are determined using equations (15) and (14), respectively.

When $Z_s/Z_b \leq 1$ (Figure 10(a) and (b)), a single stress pulse appears in the bar as well as the striker. The striker is stress-free after the primary pulse. These features are consistent with Figures 4 and 5, respectively. When $Z_s/Z_b > 1$ (Figure 9(c)), a series of step-wise residual stress pulses follows the primary pulse, resulting from the multiple cycles of impact (Figures 7 and 8).

Numerical verification

Particle velocity. To verify the results of the 1D analyses of the particle velocity (Figure 9) in the '1D prediction' section, numerical simulations of the striker impact on bars with different cross-sectional areas were carried out. The node velocities were extracted at the impact surfaces of the striker and bar, which are presented in Figure 11. In this figure, the velocities of nodes located at the impact surfaces of the striker and bar start to change from 15 and 0 m/s, respectively, after 6.667 μ s, which is the required time for the

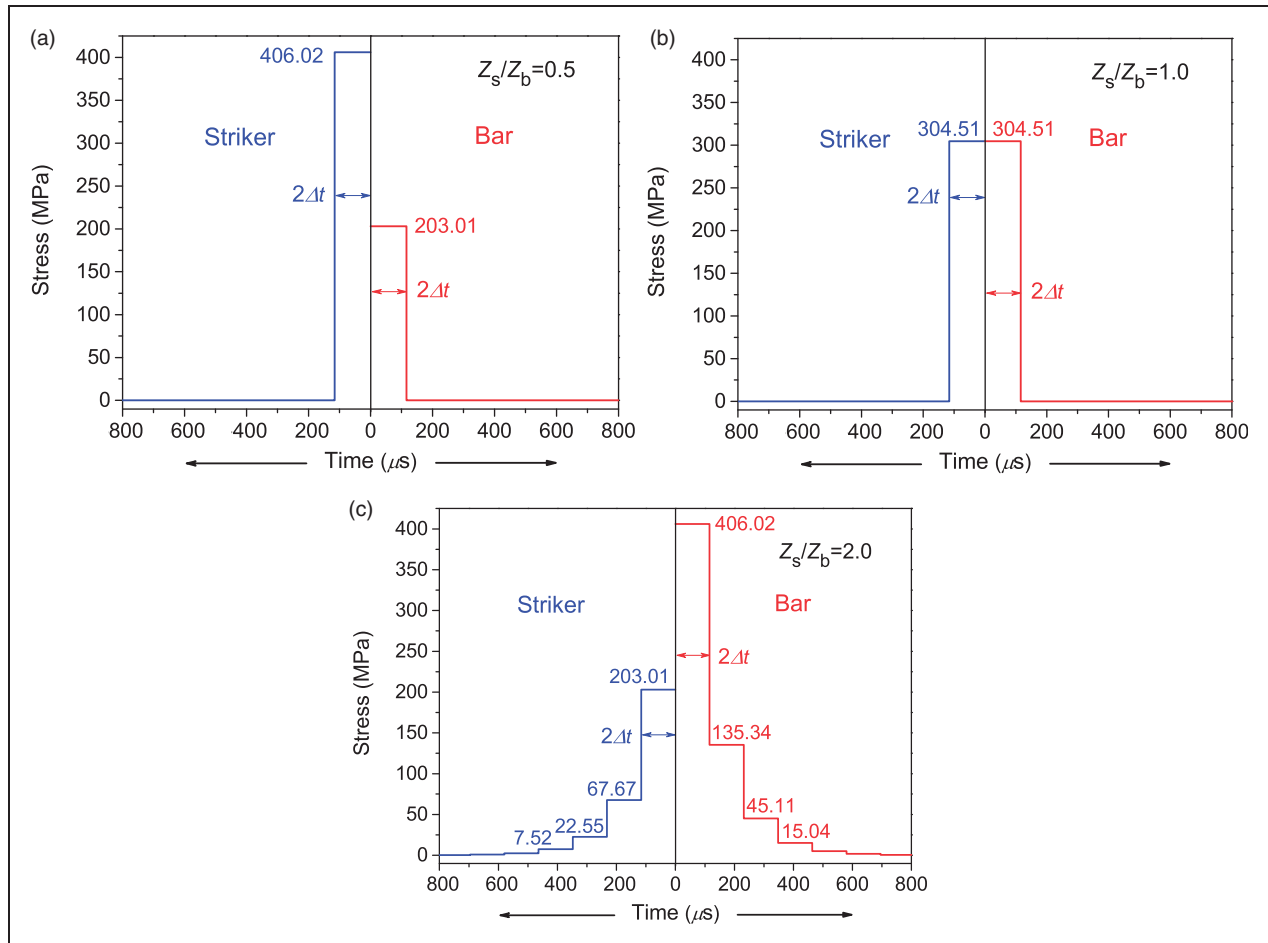


Figure 10. Stresses at impact surfaces of striker and bar. Δt is one-way travel time of elastic wave in striker. Impact velocity was 15 m/s, and Z_s/Z_b ratios were: (a) 0.5, (b) 1.0 and (c) 2.0.

striker to travel the initial gap (0.1 mm) between the striker and bar.

In Figure 11, the plateau region of the primary particle velocity pulse is highly oscillating unlike the 1D prediction result (Figure 9). Such oscillation in the simulation originates from the inertia in the radial expansion of the bar and striker, which occurs because the considered impact system is a pseudo-1D system instead of a purely 1D system. The radial expansion of the bar/striker is associated with the axial movement of particles.^{9,44}

For the case where $Z_s/Z_b = 0.5$ (Figure 11(a)), the initial particle velocity of the striker (15 m/s) decreases to 5 m/s upon impact at $6.667 \mu s$. This particle velocity value is maintained during the primary pulse period ($2\Delta t$). At $t \geq 2\Delta t$, the striker particle velocity finally changes to -5 m/s, indicating a backwards translation. All these features are consistent on a quantitative base with the 1D analysis result presented in Figure 9(a).

The simulation result in the case of $Z_s/Z_b = 1$ (Figure 11(b)) is also consistent with the prediction result (Figure 9(b)) on a quantitative base: a U_p value of 7.5 m/s during the primary pulse period and zero thereafter at both the striker and bar. The striker

particle velocity is highly fluctuating after the primary pulse period. Its consequence is described later in Figure 12(b).

The predicted magnitudes of the particle velocities for the case where $Z_s/Z_b = 2.0$ are shown in Figure 9(c). In Figure 9(c), the 1D theory-predicted particle velocities in the first four cycles were 10.00, 3.33, 1.11 and 0.37 m/s in order from the first pulse. In Figure 10(c), these values are superposed to the simulated particle velocity profile using dashed horizontal lines. The predicted values are quantitatively consistent with the simulation results, confirming the phenomenon of multiple impact cycles predicted in the 1D analysis.

Displacement of the impact surfaces. To further verify the phenomenon of separation or contact of the impact surfaces depending on the Z_s/Z_b ratio, displacements of the nodes at the impact surfaces of the striker and bar were extracted in the numerical simulation. The results are shown in Figure 12. When preparing Figure 12, the travel distance (0.1 mm) of the striker before impacting the bar was subtracted from the displacement of the striker node. The subtracted result is presented in Figure 12.

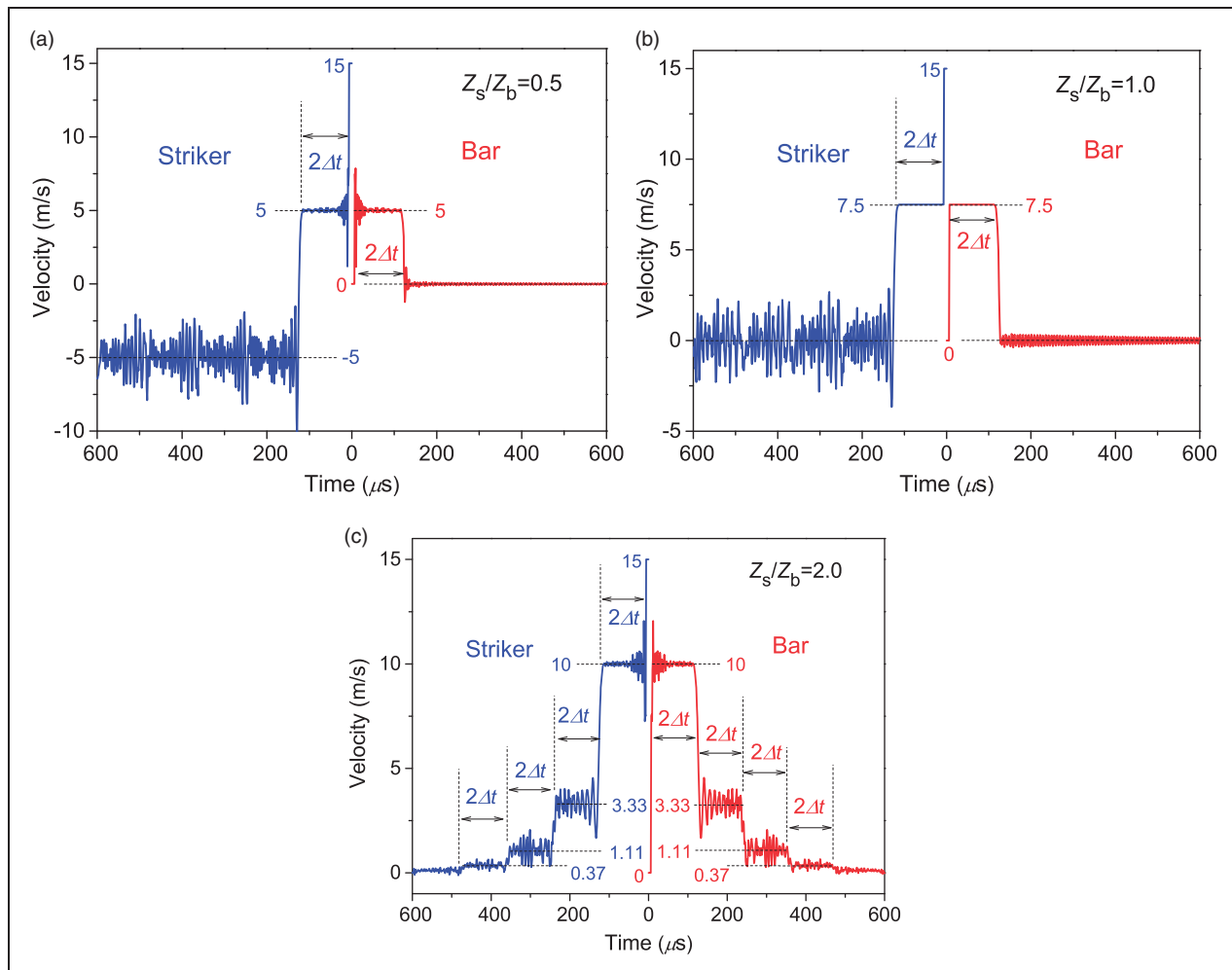


Figure 11. Simulation results of particle velocity-time curve monitored at nodes on impact surfaces of striker and bar. Impact velocity was 15 m/s and Z_s/Z_b ratios were: (a) 0.5, (b) 1.0 and (c) 2.0.

When $Z_s/Z_b = 0.5$ (Figure 12(a)), the striker is separated after the primary pulse period. This observation is consistent with the results of 1D prediction in Figure 9(a) and simulation in Figure 11(a), both of which indicated the backwards translation of the striker (negative particle velocity) at $t > 2\Delta t$.

When $Z_s/Z_b = 1.0$, a stationary striker state was anticipated at $t > 2\Delta t$ in the 1D prediction (Figure 9(b)); the particle velocity of the whole striker region was anticipated to be zero. In the numerical simulation result shown in Figure 12(b), however, the striker is separated slightly from the bar after the primary pulse period ($2\Delta t$). This slight separation of the striker is ascribed to the highly fluctuating nature of the particle velocity in the striker after the pulse time (Figure 11(b)), which results from the pseudo-1D nature of the striker-bar impact system as opposed to a purely 1D system.

In the case where $Z_s/Z_b = 2.0$ (Figure 12(c)), the two displacement curves are perfectly coincident. Striker and bar surfaces move simultaneously without

separation; the striker compresses the bar continually even after the primary pulse period ($2\Delta t$). In the post processing stage of the numerical simulation, the contact (impact) surface was also visually observed at a number of time steps. It was observed that, although the speed of the striker decreased at the moment a new cycle started, the striker was never separated from the bar but compressed the bar continually (the result of the visual observation is not shown here). Therefore, it is confirmed that, at the start of a new impact cycle, the striker is not separated from the impact surface, but continually compresses the bar particles, so that the particle velocity of the new cycle given by equation (26) ($U_{p,k} = \left(\frac{Z_s}{Z_s+Z_b}\right)\left(\frac{Z_s-Z_b}{Z_s+Z_b}\right)^{k-1} V$) is achieved (Figures 9(c) and 11(c)).

Because the striker is not separated from the impact surface (Figure 12(c)), the term ‘multiple cycles of impact’ or ‘multiple impact cycles’ used in this study excludes the case of the ‘multiple impacts (secondary impacts)’ where the striker is separated

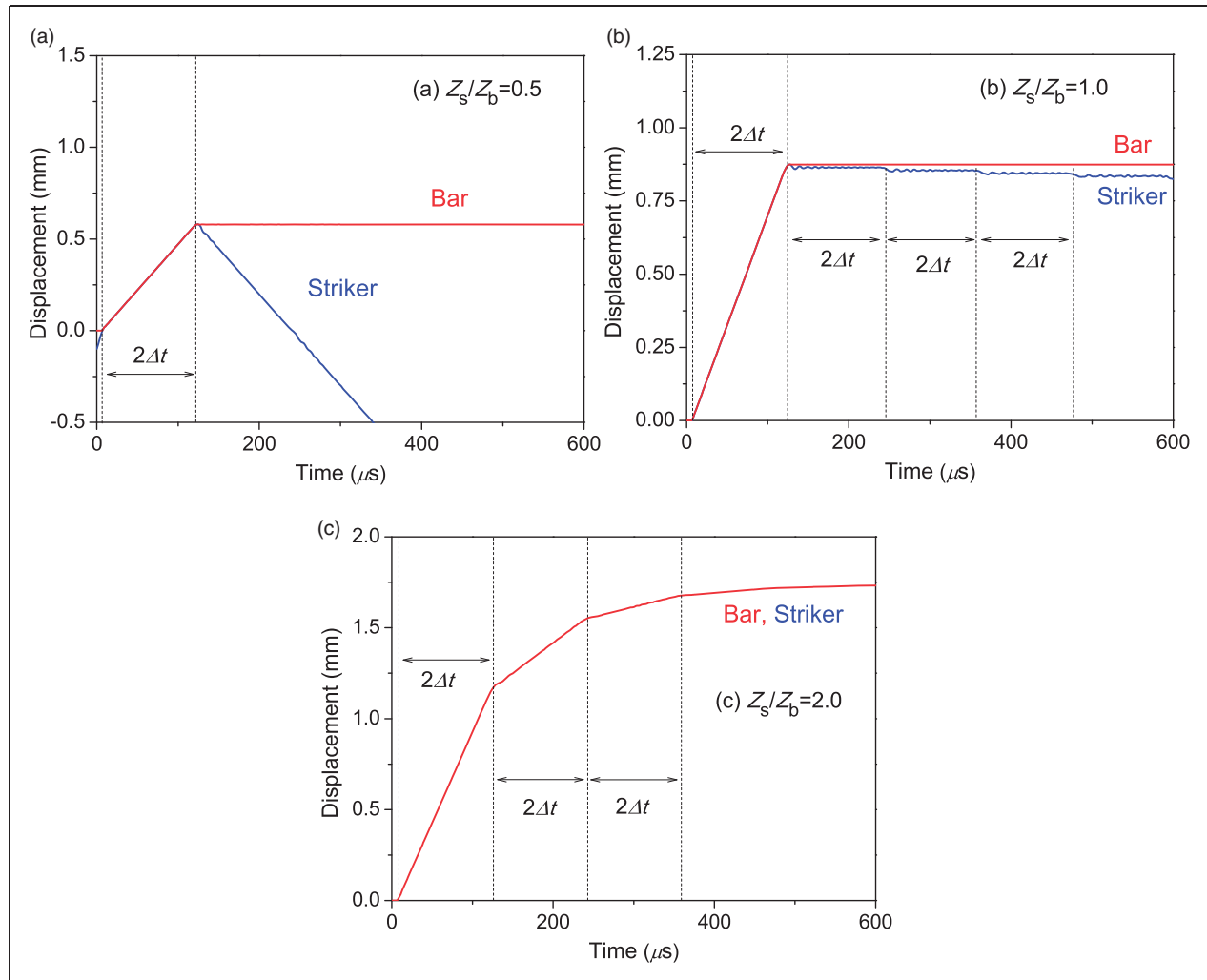


Figure 12. Simulation results of displacement-time curve monitored at nodes on impact surfaces of striker and bar. Impact velocity was 15 m/s, and Z_s/Z_b ratios were: (a) 0.5, (b) 1.0 and (c) 2.0.

from the impact surface and re-impacts the bar. However, in the case of the split Hopkinson tension bar,⁴² contrary to the compressive impact on a simple bar considered in this study, ‘multiple impacts (secondary impacts)’ can take place when the general impedance of the striker is higher than the flange; the striker can be separated from the impact surface and re-impacts the flange. According to Shin et al.,⁴¹ the secondary impacts at the striker/flange impact surface result from the difference in flange geometry from the simple bar considered in this study.

In Figure 12(c) ($Z_s/Z_b = 2.0$), the slope of the displacement curve changes after every $2\Delta t$ time interval. This observation also indicates the multiple cycles of impact because the change in particle velocity at the start of a new impact cycle inevitably results in the change in the slope of the displacement–time curve.

Stress. To verify the results of the 1D analyses of the stresses in the striker and bar (Figure 10) in the ‘1D

prediction’ section, the stresses in the striker and bar were extracted in the numerical simulation from elements at 30 mm (L') from the impact surface, and the results are shown in Figure 13. In Figure 13, the stress pulse period monitored in the striker are slightly shorter than that in the bar because the pulse duration time in the striker is $2(L - L')/C$ while it is $2L/C$ in the bar, where $C = C_s = C_b$, L is the striker length and L' is the distance of the stress-extracting elements from the impact surface.

As can be seen in Figure 13, as per the particle velocity in Figure 10, only the primary stress pulse appears when $Z_s/Z_b \leq 1$ while a series of step-wise residual stress pulses follow the primary pulse when $Z_s/Z_b > 1$. In Figure 13, all the predicted values in Figure 10 based on the 1D theory are superimposed to the simulation results using dashed horizontal lines. All the simulation results in Figure 13 are consistent with the 1D analysis results on a quantitative base. This observation also numerically verifies the 1D theory formulated in this study.

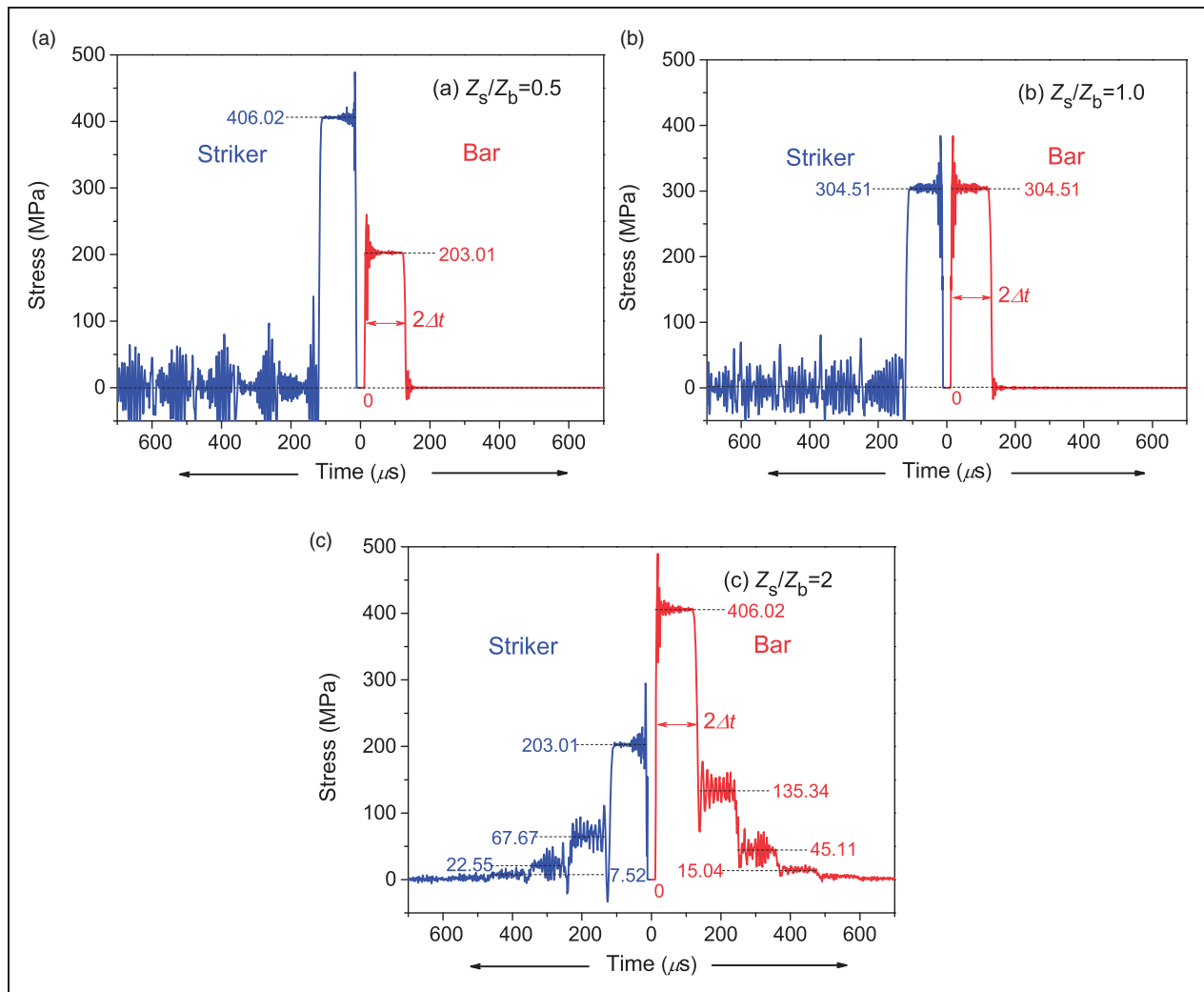


Figure 13. Simulation result of stress–time curve monitored at elements in striker and bar. Impact velocity was 15 m/s, and Z_s/Z_b ratios were: (a) 0.5, (b) 1.0 and (c) 2.0.

Conclusions

Impacts of a striker on a bar with different general impedance ($Z = A\rho C$, where A is the cross-sectional area, ρ is the density and C is the 1D speed of the elastic wave) were analysed based on 1D assumptions. The relationships between the particle velocity (U_p) and stress (σ) in both the striker and bar were formulated first by considering Newton's second law. The particle velocity and stress were then expressed in terms of the impact velocity (V) by applying momentum conservation before and after striker impact. The particle velocity and stress of the release wave, which is generated at the rear end of the striker and propagates towards the striker front, were also formulated as a function of the impact velocity by considering the release of the compressed state of the particles in the striker. As a result, the particle velocity and stress of the striker and bar could be calculated up to the time when the release wave reached the striker front. Departure of the impact-generated stress wave towards the rear end

of the striker and arrival of the release wave from the rear to the front of the striker constitutes an impact cycle.

In cases where $Z_s \leq Z_b$ (subscripts 's' and 'b' denote striker and bar, respectively), only one impact cycle takes place because the release wave was tensile or zero, and therefore, striker is separated or stationary after the first impact cycle. As a result, only a single (primary) pulse is observed in the bar and striker. In case where $Z_s > Z_b$, however, multiple impact cycles occur because the release wave is compressive, and the striker compresses the bar continually after the first cycle. As a result, a series of step-wise residual pulses follow the primary pulse in the bar and striker. The magnitudes of the stress and particle velocity in the bar and striker were quantitatively consistent with the results of the numerical analyses, verifying the 1D equations formulated in this study. The formulated equations could be useful for better understanding the various wave interaction phenomena that occur in a pseudo-1D impact system and for modifying/designing an impact system.

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ORCID iD

Hyunho Shin  <https://orcid.org/0000-0002-6740-0522>

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